

Polegate School

Maths Calculation Policy

2019-2020

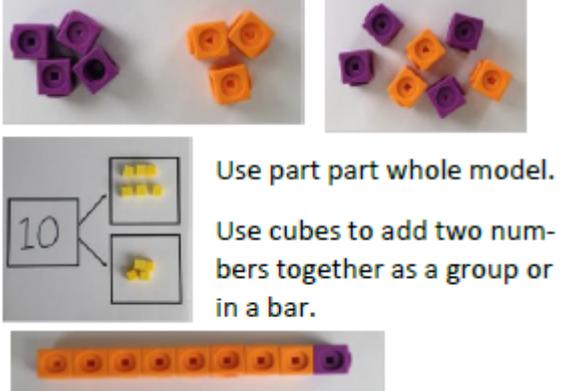
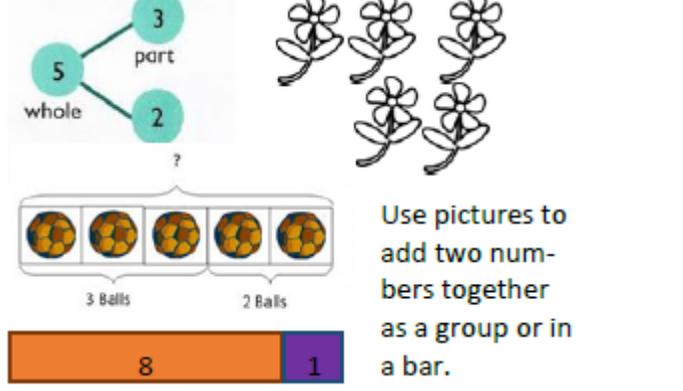
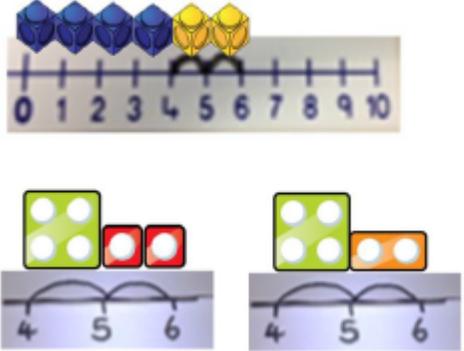
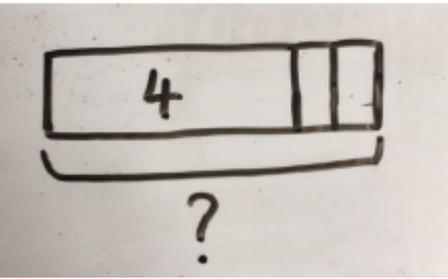
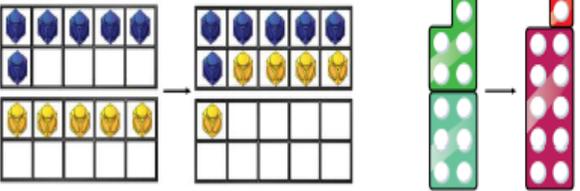
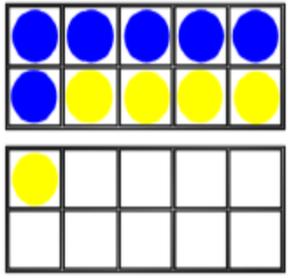


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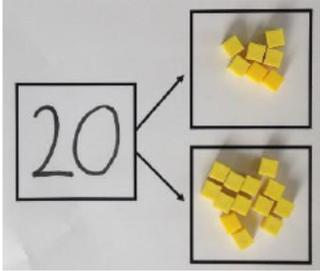
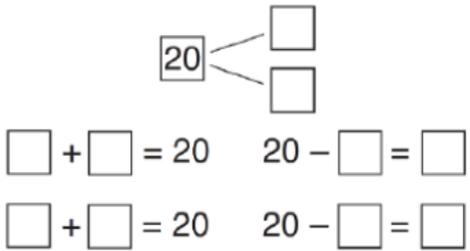
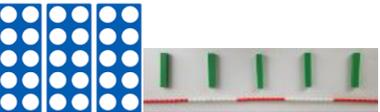
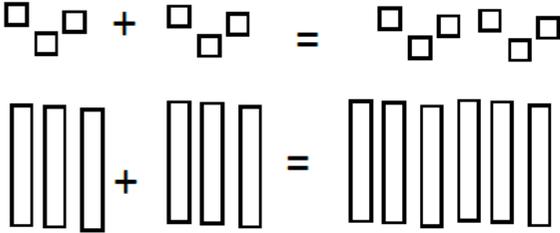
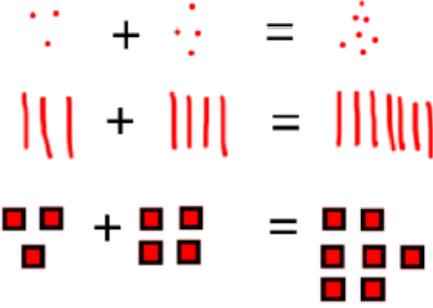
Addition

Year 1	Year 2	Year 3
Expected	Expected	Expected
<ul style="list-style-type: none"> • read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • add and subtract one-digit and two-digit numbers to 20, including 0 • Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = ? - 9$ 	<ul style="list-style-type: none"> • solve problems with addition and subtraction: <ul style="list-style-type: none"> ○ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ○ applying their increasing knowledge of mental and written methods • recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> ○ a two-digit number and 1s ○ a two-digit number and 10s ○ 2 two-digit numbers ○ adding 3 one-digit numbers • show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems 	<ul style="list-style-type: none"> • add and subtract numbers mentally, including: <ul style="list-style-type: none"> a three-digit number and 1s a three-digit number and 10s a three-digit number and 100s • add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> • memorise and reason with number bonds to 10 and 20 in several forms (for example, $9 + 7 = 16$; $16 - 7 = 9$; $7 = 16 - 9$). • realise the effect of adding or subtracting 0. • establish addition and subtraction as related operations. • combine and increase numbers, counting forwards and backwards. • discuss and solve problems in familiar practical contexts, including using quantities. • Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. 	<ul style="list-style-type: none"> • extend their understanding of the language of addition and subtraction to include sum and difference. • practise addition and subtraction to 20 is fluent in deriving facts such as using $13 + 7 = 20$; $20 - 17 = 3$ and $20 = 17 + 3$ to calculate $30 + 70 = 100$; $100 - 70 = 30$ and $70 = 100 - 30$. • check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). This establishes commutativity and associativity of addition. • record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers. • $14 + \underline{\quad} = 15 + 27$ Use the inverse to work out difficult problems. 	<ul style="list-style-type: none"> • practise solving varied addition and subtraction questions. • for mental calculations with two-digit numbers, the answers could exceed 100. <p>use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent</p>

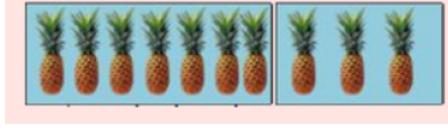
Year 1 – CPA Approach: Addition

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole part: part-whole model.</p>	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>$10 = 6 + 4$</p>
<p>Counting on using number lines.</p>	<p>Counting on using number lines using cubes or Numicon.</p> 	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line:</p> <p>What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 
<p>Regrouping to make 10</p>	<p>Regrouping to make 10; using ten frames and counters/cubes or using Numicon.</p> <p>$6 + 5$</p> 	<p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to develop an understanding of equality e.g.</p> <p>$6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$</p>

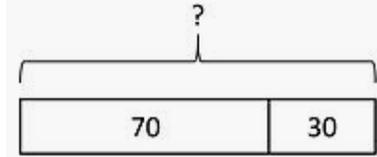
Year 2 - CPA Approach: Addition

Objective and Strategy	Concrete	Pictorial	Abstract
Use known facts: part, part whole.	 <p>Children explore ways of making numbers within 20</p>	 <p>□ + □ = 20 20 - □ = □ □ + □ = 20 20 - □ = □</p>	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Adding multiples of ten	<p>30 + 20 = 50 50 = 20 + 30</p> 	 <p>3 tens + 5 tens = _____ tens 30 + 50 = _____</p> <p>Use representations for base ten.</p>	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Adding multiples of ten: Use known facts		 <p>Children draw representations of H,T and O</p>	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$

Adding multiples of ten: Bar model



$$7 + 3 = 10$$



$$70 + 30 = 100$$

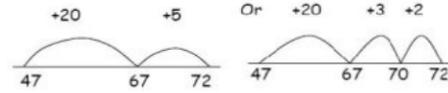
Year 2 - CPA Approach: Addition

Objective and strategy	Concrete	Pictorial	Abstract
<p>Add a two digit number and tens.</p>	<p>$25 + 10 = 35$</p> <p>Explore why the ones digit doesn't change.</p>	<p>$27 + 30$</p> <p>+10 +10 +10</p> <p>27 37 47 57</p> <p>Differentiation: either use real numbers lines or children to draw their own empty number lines.</p>	<p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p>
<p>Add a two digit number and ones.</p> <p>The 'making ten first' strategy.</p>	<p>$17 + 5 = 22$</p> <p>Use ten frame to make 'magic ten'</p> <p>Children explore the pattern.</p> <p>$17 + 5 = 22$</p>	<p>Use part part whole and number line to model.</p> <p>$17 + 5 = 22$</p> <p>16 20 23</p>	<p>$17 + 5 = 22$</p> <p>Explore related facts</p> <p>$17 + 5 = 22$</p> <p>$5 + 17 = 22$</p> <p>$22 - 17 = 5$</p> <p>$22 - 5 = 17$</p>

Add a two 2 digit numbers.



Model using dienes , place value counters and numicon



Use number line and bridge ten using part whole if necessary.

$$\begin{array}{r} 25 + 47 \\ \swarrow \quad \searrow \\ 20 + 5 \quad 40 + 7 \end{array}$$

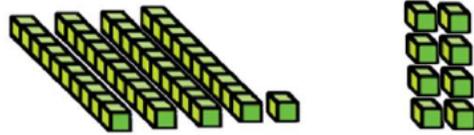
$$20 + 40 = 60$$

$$5 + 7 = 12$$

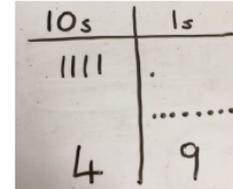
$$60 + 12 = 72$$

Add a two 2 digit numbers.

$$41 + 8$$

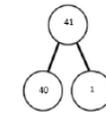


Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.

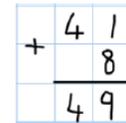


Use the formal method only if children are secure with their mental strategies.

$$41 + 8$$

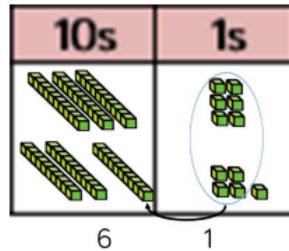


$$\begin{array}{l} 1 + 8 = 9 \\ 40 + 9 = 49 \end{array}$$

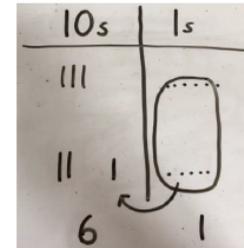


Continue to develop partitioning and place value: TO + TO

$$36 + 25$$



Children to represent the base 10 in a place value chart.



Only move onto the formal method if children are secure with their mental strategies of adding TO + TO.

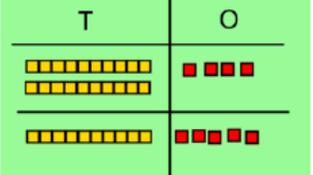
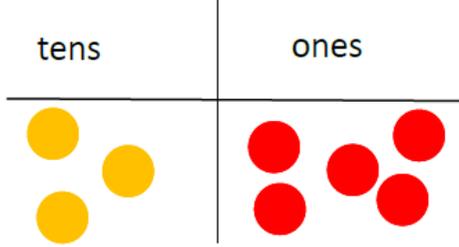
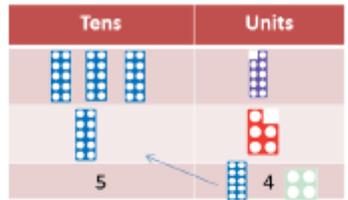
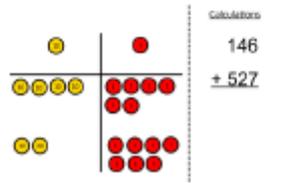
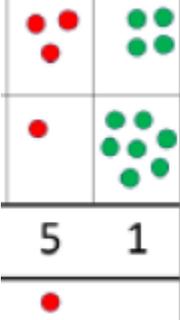
Looking for ways to make 10.

$$\begin{array}{r} 36 + 25 = \\ \swarrow \quad \searrow \\ 1 \quad 5 \end{array} \quad \begin{array}{l} 30 + 20 = 50 \\ 5 + 5 = 10 \\ 50 + 10 + 1 = 61 \end{array}$$

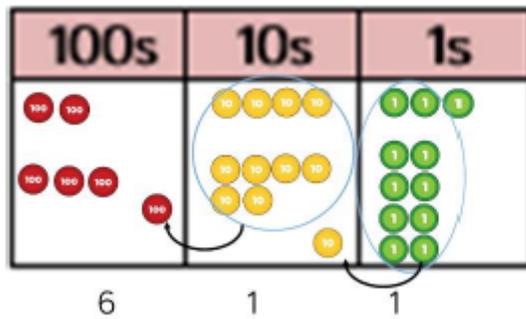
Formal method:

$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ \hline 1 \end{array}$$

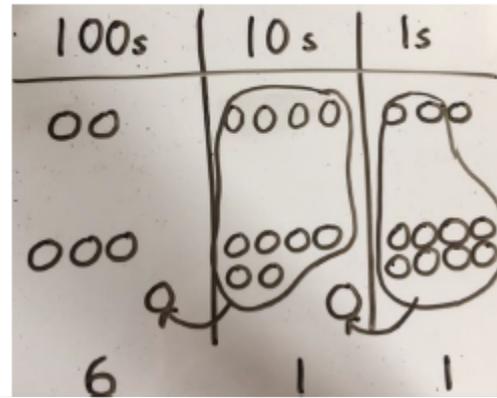
Year 3 – CPA Approach: Addition

Objective and strategy	Concrete	Pictorial	Abstract												
<p>Add numbers up to 3 digits using a formal written method</p> <p>Column Addition – No regrouping</p>	<p>Model using Dienes or numicon</p>  <p>Add together the ones first, then the tens.</p> <table border="1" data-bbox="380 462 795 686"> <thead> <tr> <th></th> <th>Tens</th> <th>Units</th> </tr> </thead> <tbody> <tr> <td>45</td> <td></td> <td></td> </tr> <tr> <td>34</td> <td></td> <td></td> </tr> <tr> <td></td> <td>7</td> <td>9</td> </tr> </tbody> </table>		Tens	Units	45			34				7	9	<p>Children move to drawing the counters using a tens and one frame.</p> 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
	Tens	Units													
45															
34															
	7	9													
<p>Add numbers up to 3 digits using a formal written method</p> <p>Column Addition – Regrouping</p>	<p>Exchange ten ones for a ten. Model using numicon and pv counters.</p>  <p>Exchange ten ones for a ten. Model using numicon and pv counters.</p>  <p>Calculations</p> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	<p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p> 	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$												

Add numbers up to 3 digits using a formal written method



Children to represent the counters in a place value chart, circling when they make an exchange.

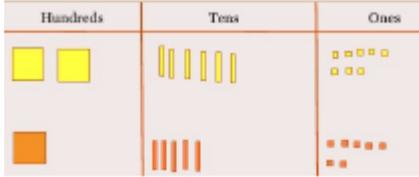
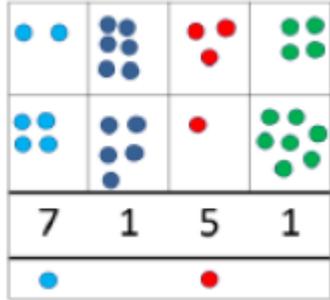
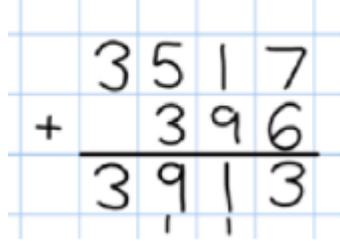


$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

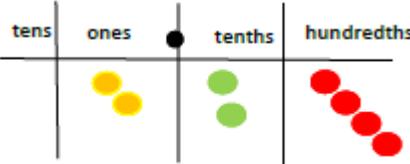
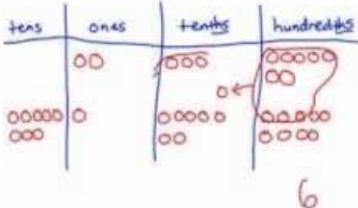
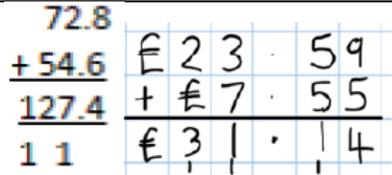
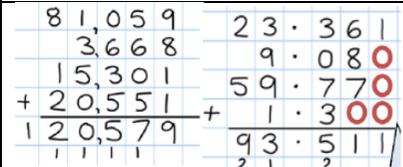
Addition

Year 4	Year 5	Year 6
Expected	Expected	Expected
<ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why round decimals with one decimal place to the nearest whole number compare numbers with the same number of decimal places up to two decimal places solve simple measure and money problems involving fractions and decimals to two decimal places. 	<ul style="list-style-type: none"> add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) add and subtract numbers mentally with increasingly large numbers use rounding to check answers to calculations and determine, in the context of a problem, accuracy solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving number up to three decimal places recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25. 	<ul style="list-style-type: none"> perform mental calculations, including with mixed operations and large numbers identify common factors, common multiples and prime numbers use their knowledge of the order of operations to carry out calculations involving the 4 operations solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency 	<ul style="list-style-type: none"> practise using the formal written methods of columnar addition with increasingly large numbers to aid fluency. practise mental calculations with increasingly large numbers, e.g., $12,462 - 2,300 = 10,162$). introduce the language of algebra as a means of solving a variety of problems mentally add and subtract tenths, and one-digit whole numbers and tenths. practise adding and subtracting decimals, including a mix of whole numbers and decimals, with different numbers of decimal places, and complements of 1 (for example, $0.83 + 0.17 = 1$). Pupils should go beyond the measurement and money models of decimals to solving puzzle 	<ul style="list-style-type: none"> practise addition for larger numbers, using the formal written methods of columnar addition undertake mental calculations with increasingly large numbers and more complex calculations. round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures. explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$. Understand common factors can be related to finding equivalent fractions develop the language of algebra as a means of solving a variety of problems

Year 4 - CPA

Objective and strategy	Concrete	Pictorial	Abstract
<p>Add numbers with up to 4 digits using the formal written methods of column addition.</p>	<p>Children continue to use dienes or place value counters to add, exchanging tens ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 		 <p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures.</p>

Year 5 & 6 - CPA

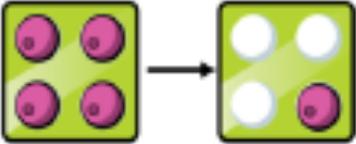
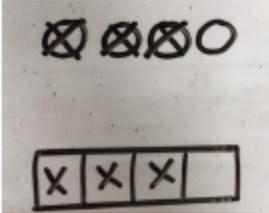
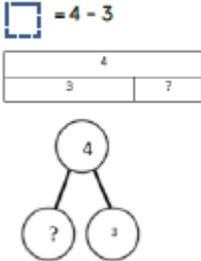
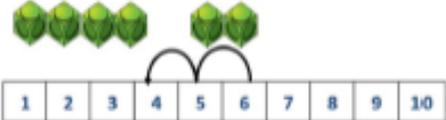
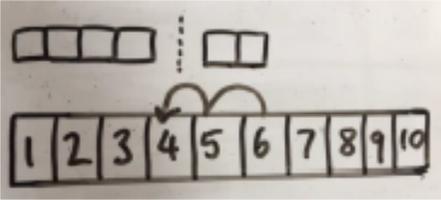
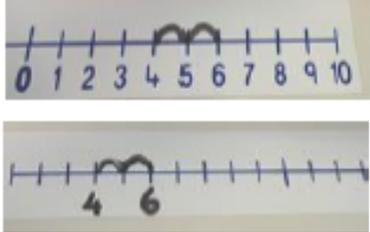
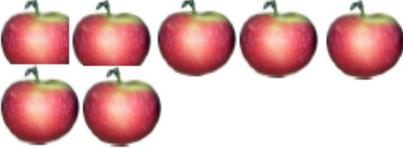
Objective and strategy	Concrete	Pictorial	Abstract
<p>Year 5</p> <p>Add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	
<p>Year 6</p> <p>Add numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>			 <p>Insert zeros for place holders.</p>

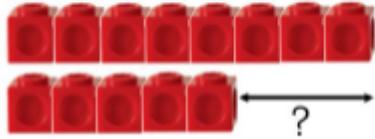
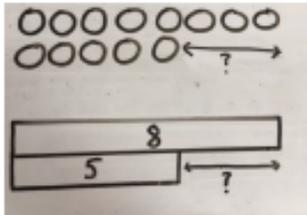
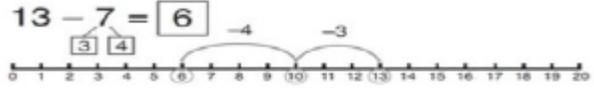
Subtraction

Year 1	Year 2	Year 3
Expected	Expected	Expected
<ul style="list-style-type: none"> count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number given a number, identify 1 more and 1 less <p>identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</p>	<ul style="list-style-type: none"> solve problems with addition and subtraction: <ul style="list-style-type: none"> using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> a two-digit number and 1s a two-digit number and 10s 2 two-digit numbers adding 3 one-digit numbers show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot <p>recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems</p>	<ul style="list-style-type: none"> add and subtract numbers mentally, including: <ul style="list-style-type: none"> a three-digit number and 1s three-digit number and 10s a three-digit number and 100s add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction estimate the answer to a calculation and use inverse operations to check answers solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> memorise and reason with number bonds to 10 and 20 in several forms (for example, $9 + 7 = 16$; $16 - 7 = 9$; $7 = 16 - 9$). realise the effect of adding or subtracting 0. This establishes addition and subtraction as related operations. combine and increase numbers, counting forwards and backwards. discuss and solve problems in familiar practical contexts, including using quantities. <p>problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.</p>	<ul style="list-style-type: none"> extend their understanding of the language of addition and subtraction to include sum and difference. practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using $3 + 7 = 10$; $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$; $100 - 70 = 30$ and $70 = 100 - 30$. check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). This establishes commutativity and associativity of addition. record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers 	<ul style="list-style-type: none"> practise solving varied addition and subtraction questions. mental calculations with two-digit numbers, the answers could exceed 100. use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent develop the language of algebra as a means of solving a variety of problems

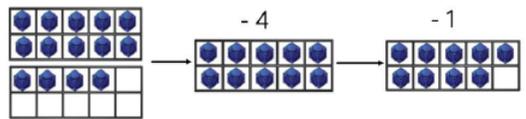
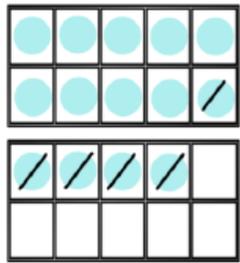
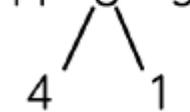
Subtraction		
Year 4	Year 5	Year 6
Expected	Expected	Expected
<ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why solve simple measure and money problems involving fractions and decimals to two decimal places. 	<ul style="list-style-type: none"> add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) add and subtract numbers mentally with increasingly large numbers use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why 	<ul style="list-style-type: none"> use their knowledge of the order of operations to carry out calculations involving the 4 operations solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy perform mental calculations, including with mixed operations and large numbers solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency 	<ul style="list-style-type: none"> practise using the formal written methods of columnar subtraction with increasingly large numbers to aid fluency. practise mental calculations with increasingly large numbers to aid fluency (for example, $12,462 - 2,300 = 10,162$). introduce the language of algebra as a means of solving a variety of problems mentally add and subtract tenths, and one-digit whole numbers and tenths. Solve difficult problems. $23 - 5 = 30 - \underline{\quad}$ 	<ul style="list-style-type: none"> practise subtraction for larger numbers, using the formal written methods of columnar subtraction, undertake mental calculations with increasingly large numbers and more complex calculations. develop the language of algebra as a means of solving a variety of problems

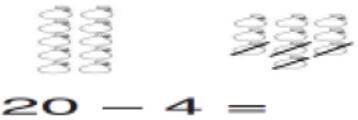
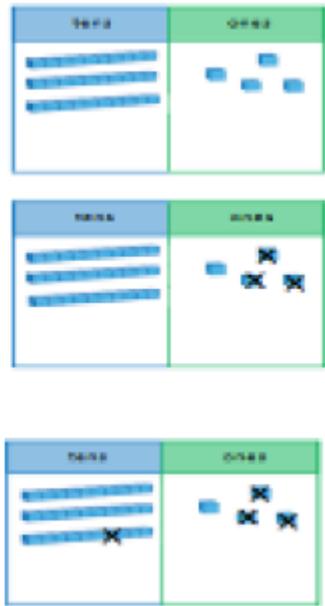
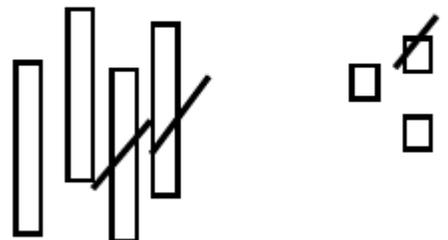
Year 1 – CPA Approach: Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract		
Subtract a one digit number.	<p>Physically taking away and removing objects from a whole.</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p>$\square = 4 - 3$</p> 		
Counting back	<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 		
Bar model	 <p>$5 - 2 = 3$</p>		<table border="1" data-bbox="1559 1031 1955 1102"> <tr> <td>8</td> <td>2</td> </tr> </table> <p>$10 = 8 + 2$</p> <p>$10 = 2 + 8$</p> <p>$10 - 2 = 8$</p> <p>$10 - 8 = 2$</p>	8	2
8	2				

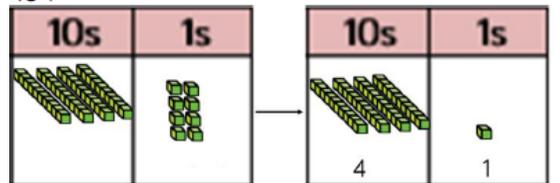
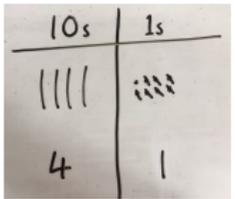
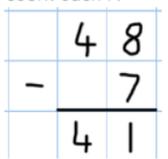
<p>Find the difference</p>	<p>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5.</p> 	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p> 	<p>Find the difference between 8 and 5.</p> <p>$8 - 5$, the difference is <input type="text"/></p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.</p>
<p>Subtraction using the 'making 10' strategy.</p>	<p>14—9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>13—7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16—8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>

Year 2 – CPA Approach: Subtraction

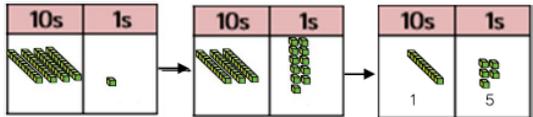
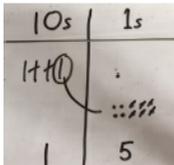
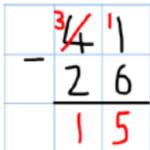
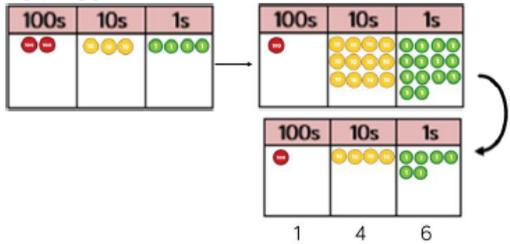
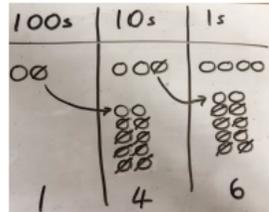
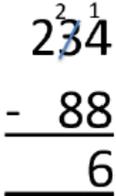
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting using the 'making ten first' strategy.</p>	<p>Making 10 using ten frames.</p> <p>$14 - 5$</p> 	<p>Children to present the ten frame pictorially and discuss what they did to make 10.</p> 	<p>Children to show how they can make 10 by partitioning the subtrahend.</p> $14 - 5 = 9$  <p>$14 - 4 = 10$ $10 - 1 = 9$</p>

<p>Regroup a ten into ten ones</p>	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 =$	$20 - 4 = 16$
<p>Partitioning to subtract without regrouping</p>	<p>$34 - 13 = 21$</p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$	$43 - 21 = 22$

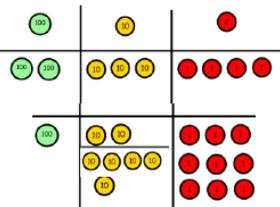
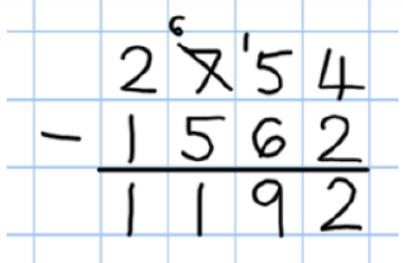
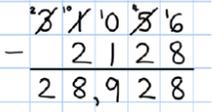
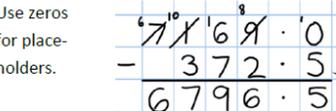
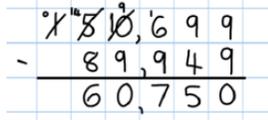
Year 2/3 – CPA Approach: Subtraction

<p>Objective & Strategy Subtract numbers including: A two digit and ones A two digit and tens Two two digits</p>	<p>Concrete Column method using base 10. 48-7</p> 	<p>Pictorial Children to represent the base 10 pictorially.</p> 	<p>Abstract Column method or children could count back 7.</p> 
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Year 3 – CPA Approach: Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtract numbers with up to three digits using a formal written method.</p> <p>Column subtraction - Regrouping</p>	<p>Column method using base 10 and having to exchange. 41 - 26</p> 	<p>Represent the base 10 pictorially, remembering to show the exchange.</p> 	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.</p> 
<p>Subtract numbers with up to three digits using a formal written method.</p> <p>Column subtraction – Regrouping</p>	<p>Column method using place value counters. 234 - 88</p> 	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p> 	<p>Formal column method. Children must understand what has happened when they have crossed out digits.</p> 

Year 4, 5 & 6 – CPA Approach: Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Year 4</p> <p>Subtract tens and ones. Subtract up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money.</i></p>	<p style="text-align: center;">$234 - 179$</p>  <p style="text-align: center;">Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw place value counters and show their regrouping – see Y3.</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p>Year 5</p> <p>Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal.</i></p>			 <p>Use zeros for place-holders.</p> 
<p>Year 6</p> <p>Subtract with increasingly large and more complex numbers and decimal values.</p>			 

Multiplication

Year 1	Year 2	Year 3
Expected	Expected	Expected
<ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher 	<ul style="list-style-type: none"> recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts 	<ul style="list-style-type: none"> count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number recognise the place value of each digit in a 3-digit number (100s, 10s, 1s) compare and order numbers up to 1,000 identify, represent and estimate numbers using different representations read and write numbers up to 1,000 in numerals and in words solve number problems and practical problems involving these ideas
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; finding simple fractions of objects, numbers and quantities make connections between arrays, number patterns, and counting in twos, fives and tens. 	<ul style="list-style-type: none"> use a variety of language to describe multiplication and division introduce the multiplication tables. practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). use inverse relations to develop reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$). 	<ul style="list-style-type: none"> use multiples of 2, 3, 4, 5, 8, 10, 50 and 100. use larger numbers to at least 1,000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, $146 = 100 + 40 + 6$, $146 = 130 + 16$). use a variety of representations, including those related to measure, pupils continue to count in 1s, 10s and 100s, so that they become fluent in the order and place value of numbers to 1,000.

Multiplication

Multiplication		
Year 4	Year 5	Year 6
Expected	Expected	Expected
<ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers recognise and use factor pairs and commutativity in mental calculations multiply two-digit and three-digit numbers by a one-digit number using formal written layout solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects 	<ul style="list-style-type: none"> identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers multiply and divide numbers mentally, drawing upon known facts multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3) solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates 	<ul style="list-style-type: none"> multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication identify common factors, common multiples and prime numbers perform mental calculations, including with mixed operations and large numbers use their knowledge of the order of operations to carry out calculations involving the 4 operations solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places multiply one-digit numbers with up to two decimal places by whole numbers solve problems which require answers to be rounded to specified degrees of accuracy recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> continue to practise recalling and using multiplication tables and related division facts to aid fluency. practise mental methods and extend this to 3-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$). practise to become fluent in the formal written method of short multiplication and short division with exact answers write statements about the equality of expressions 	<ul style="list-style-type: none"> practise and extend their use of the formal written methods of short multiplication and short division. apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. use and understand the terms factor, multiple and prime, square and cube numbers. interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as 	<ul style="list-style-type: none"> practise multiplication for larger numbers, using the formal written methods of short and long multiplication. undertake mental calculations with increasingly large numbers and more complex calculations. continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

(for example, use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$).

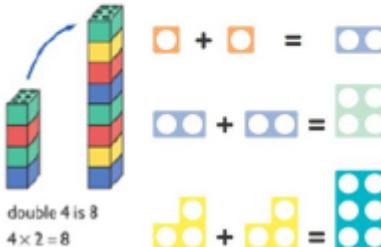
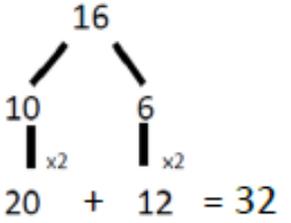
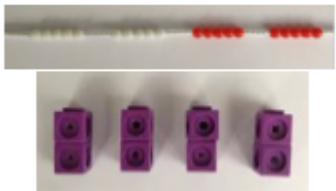
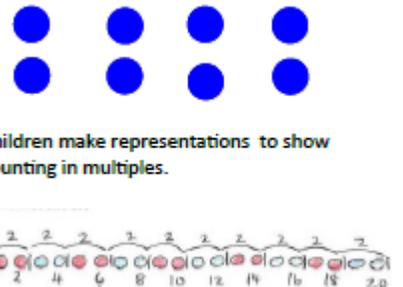
- combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, $2 \times 6 \times 5 = 10 \times 6 = 60$.
- solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children

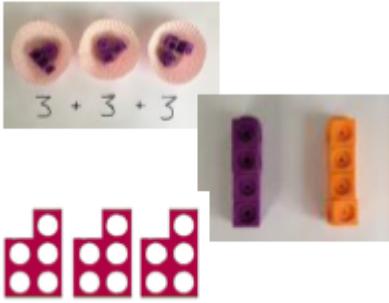
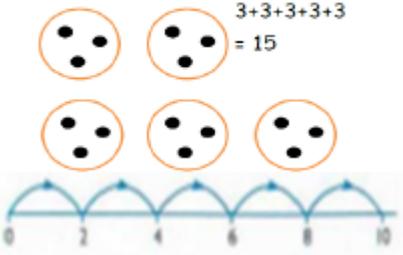
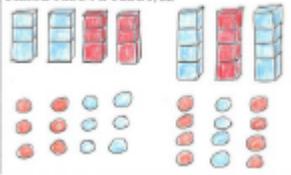
decimals or by rounding (for example, $98 \div 4 = 98/4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

- use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres.
- introduce the language of algebra as a means of solving a variety of problems
- distributivity can be expressed as $a(b + c) = ab + ac$.
- understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).
- use and explain the equals sign to indicate equivalence, including in missing number problems (for example $13 + 24 = 12 + 25$; $33 = 5 \times ?$).

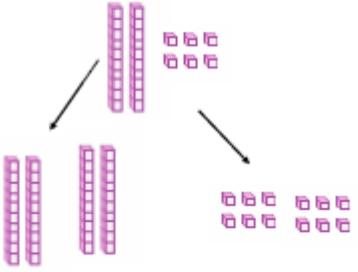
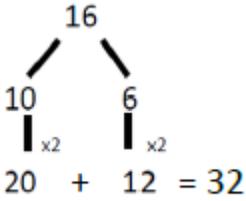
- understand common factors can be related to finding equivalent fractions
- develop the connection made between multiplication and division with fractions, decimals, percentages and ratio
- develop the language of algebra as a means of solving a variety of problems
- multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers.
- multiply decimals by whole numbers, starting with the simplest cases, such as $0.4 \times 2 = 0.8$, and in practical contexts, such as measures and money.

Year 1 – CPA Approach: Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 
<p>Counting in multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	<p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
<p>Making equal groups and counting the total.</p>	 <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p>

<p>Repeated addition</p>	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2 + 2 + 2 + 2 + 2 = 10$</p>
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	<p>$3 \times 2 = 6$</p> <p>$2 \times 5 = 10$</p>

Year 2 – CPA Approach: Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Model doubling using dienes and PV counters.</p>  <p>$40 + 12 = 52$</p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>

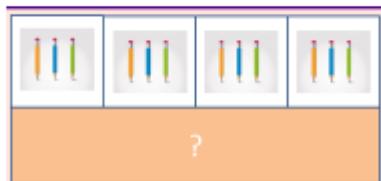
Counting in multiples of 2, 3, 4, 5, 10 from 0.

(Repeated addition)

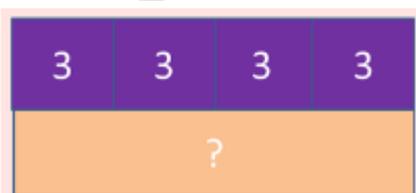
Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.



$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$$



Number lines, counting sticks and bar models should be used to show representation of counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10

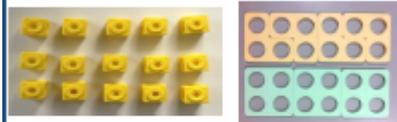
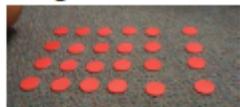
0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30

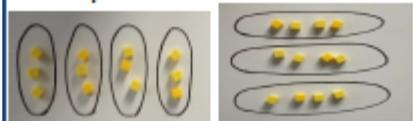
$$4 \times 3 = \square$$

Multiplication is commutative.

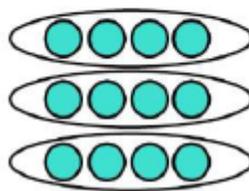
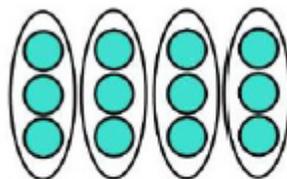
Create arrays using counters and cubes and Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

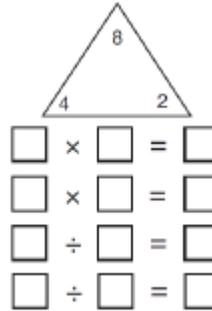
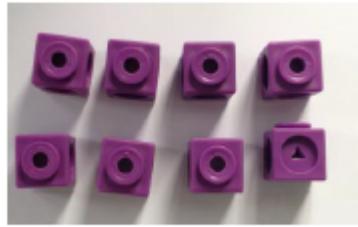
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Using the Inverse.

This should be taught alongside division, so pupils learn how they work alongside each other.



$2 \times 4 = 8$

$4 \times 2 = 8$

$8 \div 2 = 4$

$8 \div 4 = 2$

$8 = 2 \times 4$

$8 = 4 \times 2$

$2 = 8 \div 4$

$4 = 8 \div 2$

Show all 8 related fact family sentences.

Year 3 – CPA Approach: Multiplication

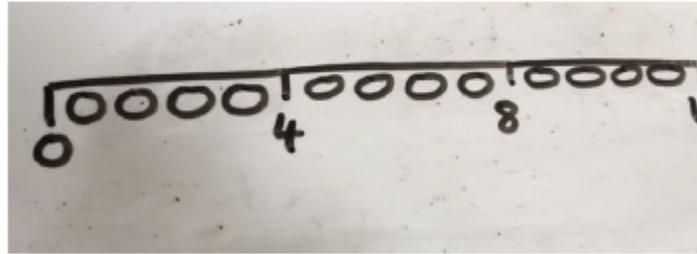
Objective & Strategy	Concrete	Pictorial	Abstract
Repeated grouping/ Repeated addition	<p>3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>

Repeated grouping/
Repeated addition

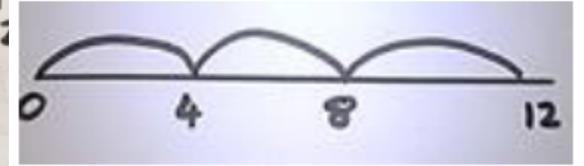
3×4



Represent this pictorially alongside a number line (Abstract number line showing three jumps of four.



$3 \times 4 = 12$

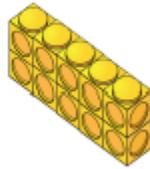


Use arrays to illustrate commutativity.

$2 \times 5 = 5 \times 2$

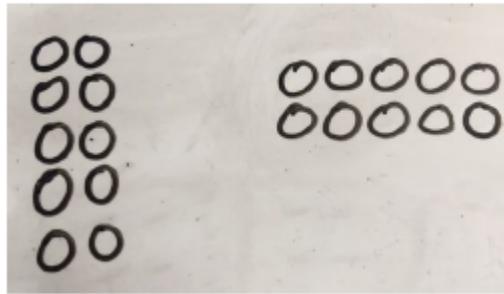


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write range of calculations e.g.

$10 = 2 \times 5$

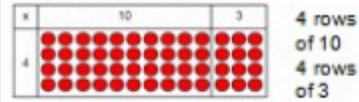
$5 \times 2 = 10$

$2 + 2 + 2 + 2 + 2 = 10$

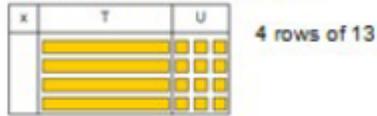
$10 = 5 + 5$

Grid method

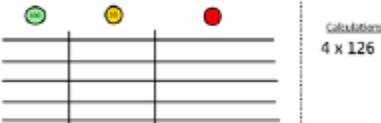
Show the links with arrays to first introduce the grid method



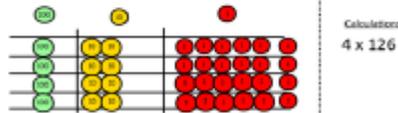
Move onto base ten to move towards a more compact method.



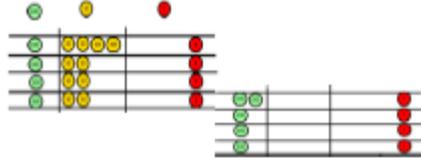
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows



Fill each row with 126



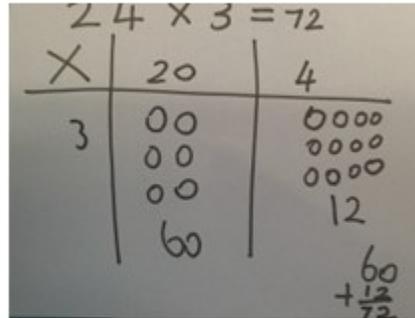
Add up each column, starting with the ones making any exchanges needed



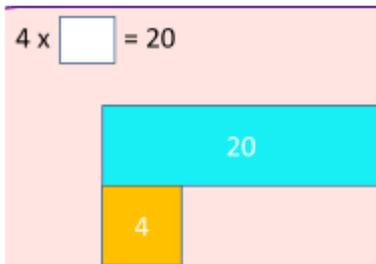
Then you have your answer.

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar models are used to explore missing numbers



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

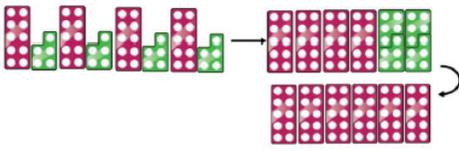
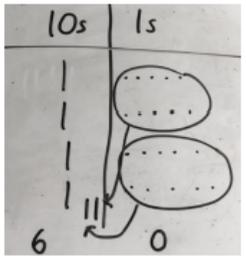
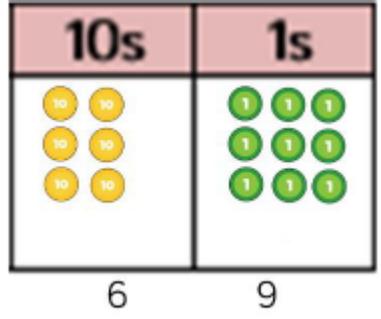
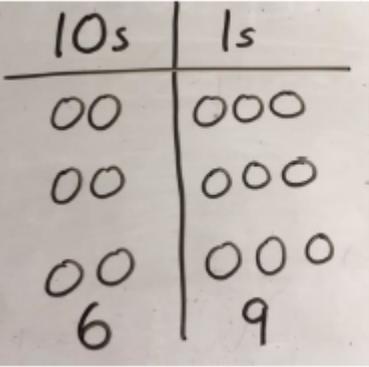
x	30	5
7	210	35

$$210 + 35 = 245$$

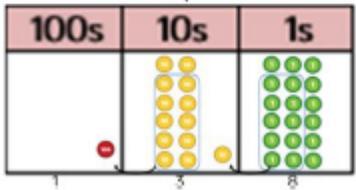
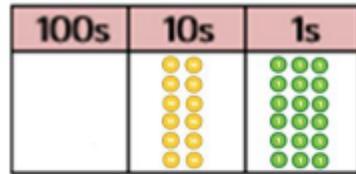
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

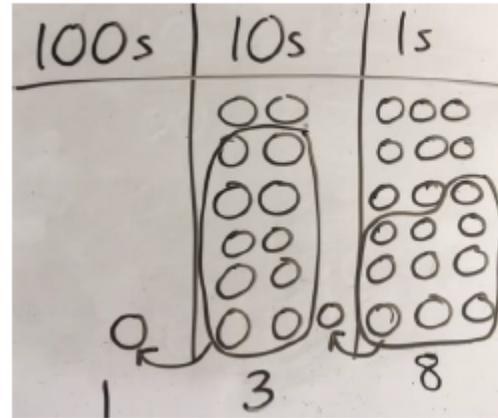
Year 4 – CPA Approach: Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Partition to multiply.	<p>4×15</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> $ \begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \\ 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array} $
Multiply a two-digit & three-digit numbers by a one digit number using a formal method.		<p>Children to represent the counters pictorially.</p> 	<p>Children to record what it is they are doing to show understanding.</p> $ \begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \\ 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array} $ $ \begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array} $

6×23



Children to represent the counters/base 10, pictorially e.g. the image below.



$6 \times 23 =$

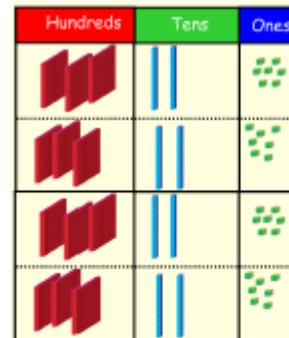
$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$$

Year 5 & 6 – CPA Approach: Multiplication

Objective & Strategy

Column multiplication for 3 and 4 digit numbers x 1 digit.

Concrete



It is important at this stage that they always multiply the ones first.

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$

Pictorial

x	300	20	7
4	1200	80	28



Abstract

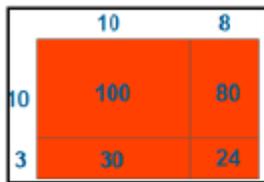
$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$

	3	2	7
x			4
	1	3	0
		2	8

This will lead to a compact method.

Column multiplication

Manipulatives may still be used with the corresponding long multiplication modelled alongside.

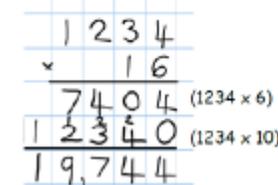


Continue to use bar modelling to support problem solving

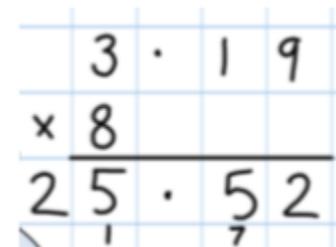


18 x 3 on the first row
(8 x 3 = 24, carrying the 2 for 20, then 1 x 3)

18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first



Year 6
Multiplying decimals up to 2 decimal places by a single digit.



Division

Year 1	Year 2	Year 3
Expected	Expected	Expected
<ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher 	<ul style="list-style-type: none"> recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts 	<ul style="list-style-type: none"> divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; finding simple fractions of objects, numbers and quantities make connections between arrays, number patterns, and counting in twos, fives and tens. 	<ul style="list-style-type: none"> use a variety of language to describe multiplication and division. begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). use commutativity and inverse relations to develop multiplicative reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$). 	<ul style="list-style-type: none"> develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (eg, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (eg, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$). develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division. solve simple problems in contexts, deciding which of the four operations to use and why. Include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (eg, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children). <p>Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)</p>

Division

Year 4	Year 5	Year 6
Expected	Expected	Expected
<ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers recognise and use factor pairs and commutativity in mental calculations find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths 	<ul style="list-style-type: none"> identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers multiply and divide numbers mentally, drawing upon known facts divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3) solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rate 	<ul style="list-style-type: none"> divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context use their knowledge of the order of operations to carry out calculations involving the 4 operations solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$] identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places use written division methods in cases where the answer has up to two decimal places solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	In-depth	In-depth
<ul style="list-style-type: none"> continue to practise recalling and using multiplication tables and related division facts to aid fluency. practise mental methods and extend this to 3-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$). practise to become fluent in the formal written method of short multiplication and short division with exact answers solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children 	<ul style="list-style-type: none"> practise and extend their use of the formal written methods of short multiplication and short division. apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. use and understand the terms factor, multiple and prime, square and cube numbers. interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$). use multiplication and division as inverses to support the introduction of ratio in year 6, for 	<ul style="list-style-type: none"> practise division for larger numbers, using the formal written methods of short and long division explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$. Understand common factors can be related to finding equivalent fractions develop the connection made between multiplication and division with fractions, decimals, percentages and ratio explore and make conjectures about converting a simple fraction to a decimal fraction (for example, $3 \div 8 = 0.375$). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to

Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)

example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres.

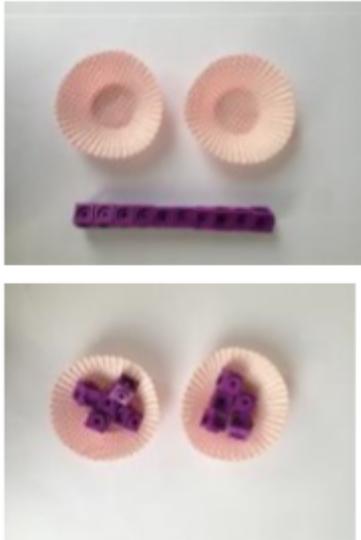
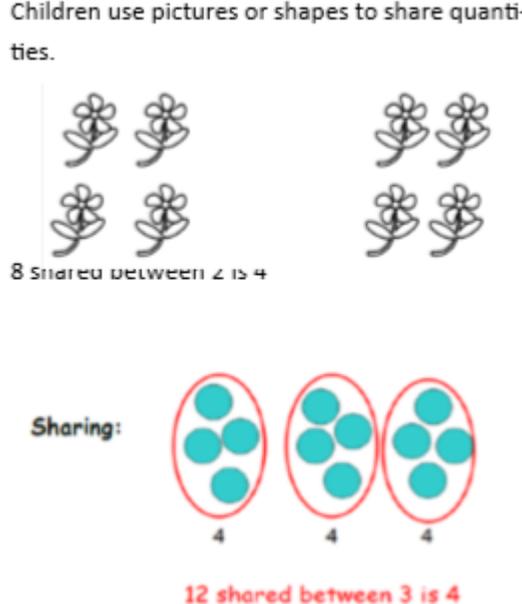
- distributivity can be expressed as $a(b + c) = ab + ac$.
- use and explain the equals sign to indicate equivalence, including in missing number problems (for example $13 + 24 = 12 + 25$; $33 = 5 \times ?$).

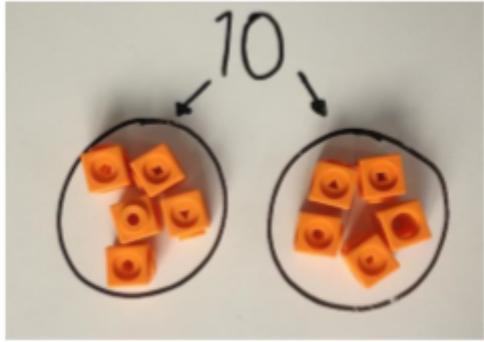
Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)

three decimal places, or other appropriate approximations depending on the context.

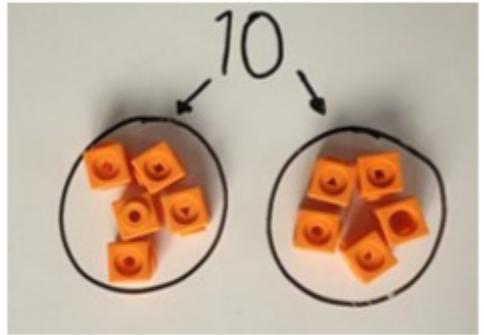
- introduce to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money.
- recognise division calculations as the inverse of multiplication.
- develop the skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations, including rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.

Year 1 & Y2 – CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Year 1 Division as sharing</p>		<p>Children use pictures or shapes to share quantities.</p>  <p>8 Shared between 2 is 4</p> <p>Sharing:</p> <p>4 4 4</p> <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is</p> <p>4</p>

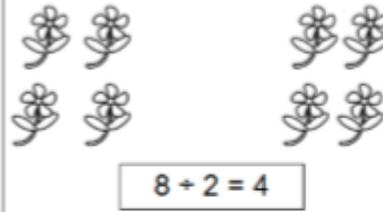


I have 10 cubes, can you share them equally in 2 groups?

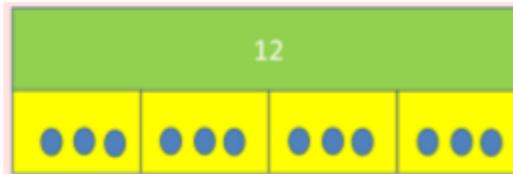


I have 10 cubes, can you share them equally in 2 groups?

Children use pictures or shapes to share quantities.



Children use bar modelling to show and support understanding.



$$12 \div 4 = 3$$

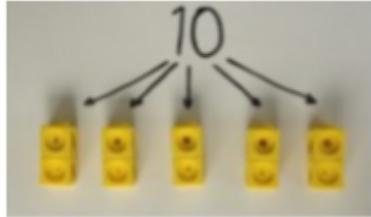
$$12 \div 3 = 4$$

Year 2
Division as sharing

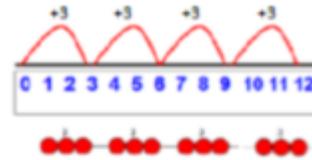
Division as grouping

Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.

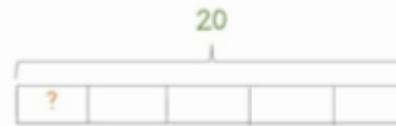


Use number lines for grouping



$$12 \div 3 = 4$$

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

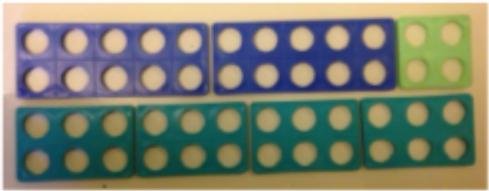
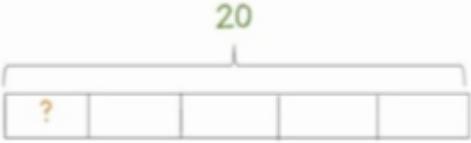
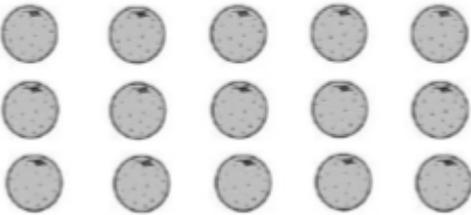


$$20 \div 5 = ?$$
$$5 \times ? = 20$$

$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

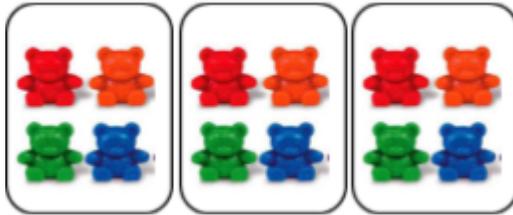
Year 3 – CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$</p> <p> $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Division with remainders.

$14 \div 3 =$

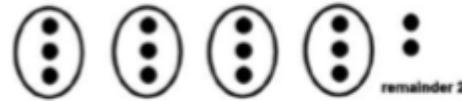
Divide objects between groups and see how much is left over



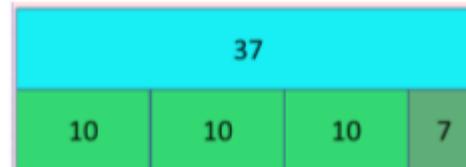
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



Use bar models to show division with remainders.

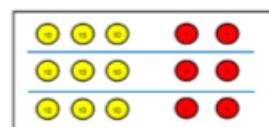
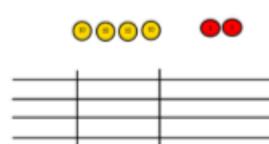
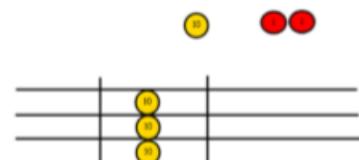
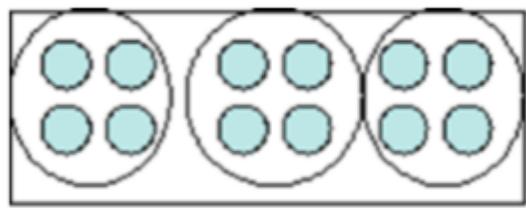
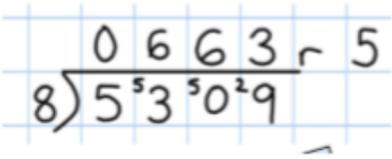


Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

↑ ↑ ↑ ↑
dividend divisor quotient remainder

Year 4 & 5- CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short division</p>	<p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>Calculations $42 \div 3$</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ 

Year 6 – Long division

Long division using place value counters

$$2544 \div 12$$

1000s	100s	10s	1s
●●	●●●●●● ●	●●●●●●	●●●●●●

We can't group 2 thousands into groups of 12 so will exchange them.

1000s	100s	10s	1s
	●●●●●● ●●●●●● ●●●●●● ●●●●●●	●●●●●●	●●●●●●

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

1000s	100s	10s	1s
	●●●●●● ●●●●●● ●●●●●●	●●●●●● ●●●●●●	●●●●●●

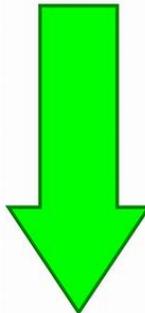
After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

1000s	100s	10s	1s
	●●●●●● ●●●●●● ●●●●●●	●●●●●●	●●●●●● ●●●●●● ●●●●●●

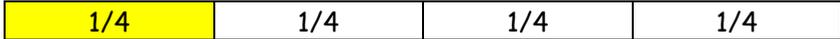
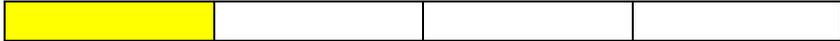
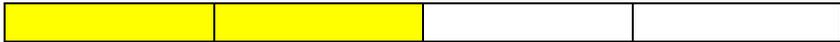
After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$



1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Fraction Policy

<u>Stage</u>	<u>National Curriculum</u>	<u>Vocabulary + Strategies</u> <u>Image</u>
Year 1	<p>N/C: recognise, find and name a half as one of two equal parts of an object, shape or quantity.</p> <p>N/C: recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</p> <p>LO - I can name and find $\frac{1}{4}$ and $\frac{1}{2}$ of a shape, an object or a quantity of objects</p>	<p><u>Shading fractions of shape</u></p> <p>Shade $\frac{1}{2}$ of this shape yellow.</p>  <p>Shade $\frac{1}{4}$ of this shape yellow</p> 
Year 2	<p>N/C: recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity.</p> <p>LO - I can find and name $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ of a length, shape, set of objects or quantity</p> <p>Greater Depth - Find and compare fractions of amounts (e.g 14 of £20 = £5 and $\frac{1}{2}$ of £8 = £4 so $\frac{1}{4}$ of £20 is greater than $\frac{1}{2}$ of £8).</p>	<p><u>Shading fractions of shape</u></p> <p>Shade $\frac{1}{3}$ of this shape yellow.</p>  <p>Shade $\frac{1}{4}$ of this shape yellow</p>  <p>Shade $\frac{2}{4}$ of this shape yellow</p>  <p>Shade $\frac{3}{4}$ of this shape yellow</p> 

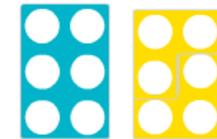
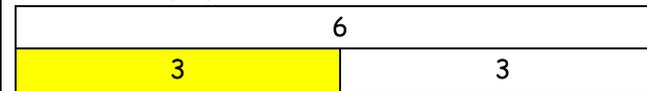
Year 2

N/C: write simple fractions e.g. $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of two quarters and one half.

LO - I can write simple fractions e.g. $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of two quarters and one half.

Recognising simple fractions

What's a half of 6?



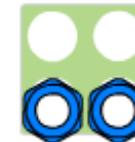
Thought Process:

For a half, divide the whole number by 2.

Recognising the equivalence of two quarters and one half



$\frac{2}{4}$ is equivalent to $\frac{1}{2}$ using Numicon



Year 3

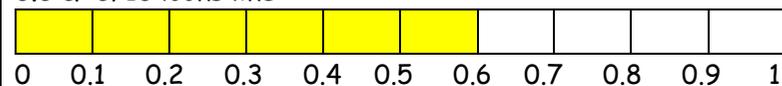
N/C: count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10

LO - I can count up and down in tenths.

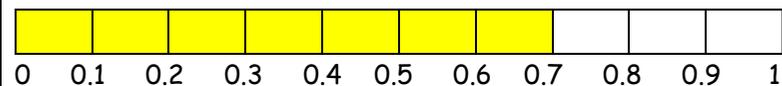
LO - I can show that tenths that arise from dividing a single digit number or a quantity by 10 are represented by a decimal number

Place value in decimal numbers

0.6 & $\frac{6}{10}$ looks like:



0.7 & $\frac{7}{10}$ looks like:



If 10 Numicon is the value of 1 then what is the value of 6? 0.6

Extend - Can you write this as a fraction? Are they the same value? How? Why?



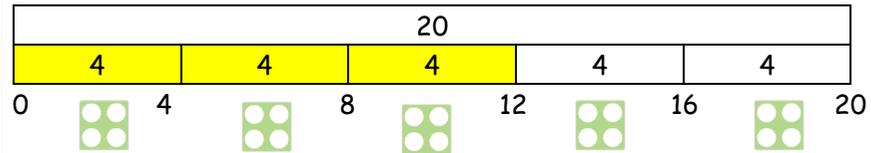
Year 3

N/C: recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.

LO - I can recognise, find and write fractions of a discrete set of objects or numbers using fractions with a small denominator or a denominator of 1 and put these in order

Fractions of an amount

Calculate $\frac{3}{5}$ of 20...



Thought process: there are 2 steps...

1. Divide the given amount by the denominator, ($20 \div 5 = 4$)
2. Multiply the answer by the numerator ($4 \times 3 = 12$)

N/C: add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)

LO- I can add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)

Adding fractions with the same denominator

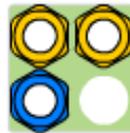
$$\frac{1}{4} + \frac{2}{4}$$

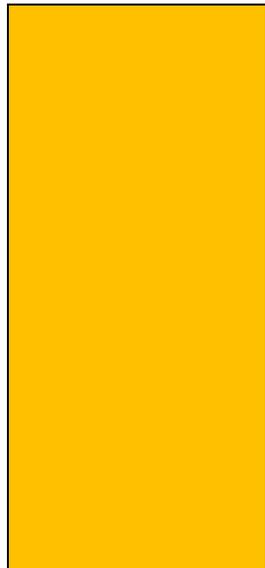


Thought Process:

As long the denominators are the same, you can add or subtract the numerators.

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$





N/C: recognise and show, using diagrams, equivalent fractions with small denominators

LO - I can recognise and show, using diagrams, equivalent fractions with small denominators

Equivalent fractions
Find equivalent fractions to $2/5$



Take each fifth and split them into two pieces



$4/10$ is therefore equivalent to $2/5$

Thought Process:
Find equivalent fractions: identify the common denominator, using knowledge of multiples and multiply the numerator by the factor used to find the common denominator, which will be different for both fraction.



N/C: recognise and show, using diagrams, families of common equivalent fractions

LO - I can recognise show and name, using diagrams, families of common equivalent fractions including tenths and hundredths

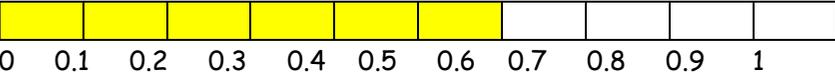
Equivalent fractions

1 whole							
$1/2$				$1/2$			
$1/4$		$1/4$		$1/4$		$1/4$	
$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$

N/C: recognise and write decimal equivalents of any number of tenths or hundredths

LO - I can count up and down in hundredths LO - I can recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten.

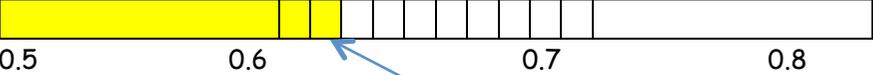
Place value in decimal numbers
0.6 looks like:



0.7 looks like:



Let's zoom in, 0.62 would look like so - it's larger than 6 but smaller than 7...



0 Ones
.
6 tenths
2 hundredths

0 . 6 2

Stage 4

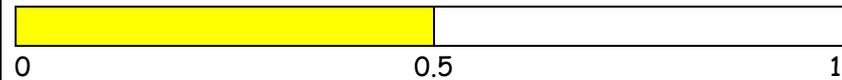
Year 4

N/C: recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$

LO - I can recognise and write decimal equivalents of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$, $\frac{n}{10}$ and $\frac{n}{100}$

Fractions to decimals and vice versa

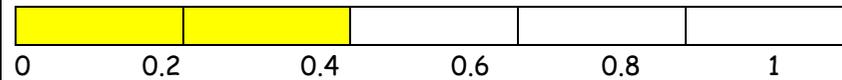
$$\frac{1}{2} = 0.5$$



$$\frac{3}{10} = 0.3$$



$$\frac{2}{5} = 0.4$$



Thought process: Divide the denominator by the numerator.

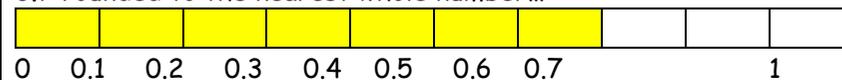
$$\frac{1}{2} \text{ as a decimal} = 2 \div 1 = 0.5$$

N/C: round decimals with one decimal place to the nearest whole number

LO- I can round decimals with one decimal place to the nearest whole number

Place value in decimal numbers - Rounding

0.7 rounded to the nearest whole number...



Thought process: we can only go to the nearest whole numbers; here they are 0 and 1.

We need to remember the rule for rounding. An easy rhyme to remember;

1, 2, 3, 4 - down to the floor. 5, 6, 7, 8, 9, - up we climb.

0.7 rounded to the nearest whole number... "5, 6, 7, 8, 9 - up we climb," we therefore will round up to 1; our nearest whole number.



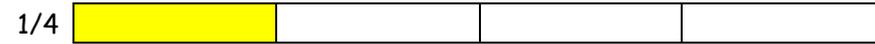
Year 4

N/C: add and subtract fractions with the same denominator

LO - I can add and subtract fractions with the same denominator

Adding fractions with the same denominator

$$1/4 + 2/4$$



+ +



= =



Stage 4 - Reverse for subtraction

Adding fractions with different denominators

$$1/3 + 2/4$$

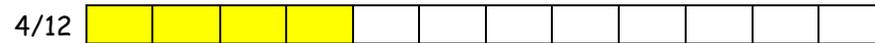


+ +



We need find a common denominator that appears in both multiplication tables...12.
Split two bars into 12

$$1/3 + 2/4 \quad \text{becomes} \quad 4/12 + 6/12$$



+ +



$$= 10/12$$



Year 5

N/C: add and subtract fractions with the same denominator and multiples of the same number.

LO- I can add and subtract fractions with the same denominator and related fractions including writing mathematical statements that exceed 1 as a mixed number: (e.g. $2/5 + 4/5 = 6/5 = 1\frac{1}{5}$)

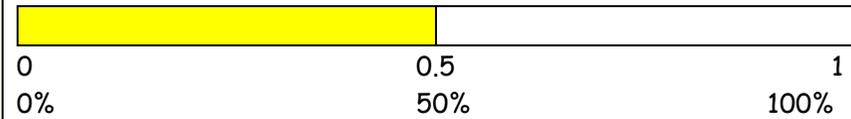
Year 5

N/C: recognise the percent symbol (%) and understand that percent relates to "number of parts per hundred", and write percentages as a fraction with denominator hundred, and as a decimal fraction

LO- I can write simple fractions as percentages and decimalized percentages (e.g. $\frac{1}{2} = 50\% = 0.5$)

Fractions to decimals to percentages

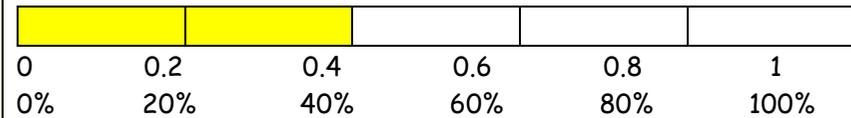
$1/2 = 0.5 = 50\%$



$3/10 = 0.3 = 30\%$

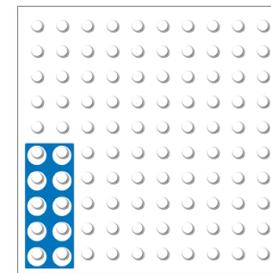
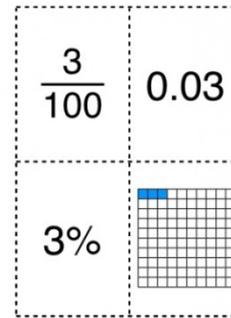


$2/5 = 0.4 = 40\%$



Thought process: Divide the denominator by the numerator and multiply by 100

$1/2$ as a decimal = $2 \div 1 = 0.5 \times 100 = 50\%$



$1/10 = 10\%$

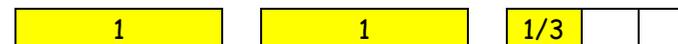
Year 5

N/C: recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements

LO- I can recognise mixed numbers and improper fractions and convert from one form to the other

Mixed numbers to improper fractions and vice versa

Convert $2 \frac{1}{3}$ into an improper fraction.



		<p>Convert these now into thirds, how many thirds are there?</p> <p style="text-align: center;"> $\frac{1}{3}$ </p> <p>= $\frac{7}{3}$</p> <p>Thought process: Multiply the whole number by the denominator, to find the improper fraction for the whole number and then add the extra numerators. e.g. $2 = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$</p>
--	--	--

Year 6	I can use common factors to simplify fractions and use common multiples to express fractions in the same denomination	<p>Thought Process: Use knowledge of multiplication tables to identify common factors to simplify fractions.</p>
	I can compare and order any fraction, including fractions >1	<p>Thought Process: To order fractions, first find equivalent fractions with a common denominator:</p> <ul style="list-style-type: none"> • Use knowledge of multiplication tables to identify common denominators (multiples). • Identify the factor with which to calculate the common denominator and then multiply the numerator by the same factor. • Order on a number line • Return to original fractions.
	I can use percentages for comparison and calculate percentages of whole numbers or measures such as 15% of 360.	<p>Thought Process: To find a percentage of given amount:</p> <ul style="list-style-type: none"> • Convert the percentage into a fraction • Divide amount given by denominator • Multiply answer by numerator

I can recall and use equivalences between simple fractions, decimals and percentages including in different contexts

Thought Process:

- To convert fractions to decimals: numerator divided by the denominator
- To convert decimals to a percentage: multiply the decimal by 100
- Convert decimals to fractions: Identify the place value of tenths, hundredths or thousandths.